Einführung

Fortgeschrittene Kapitel
der
Fluoreszenzspektroskopie
und
Fluoreszenzmikroskopie

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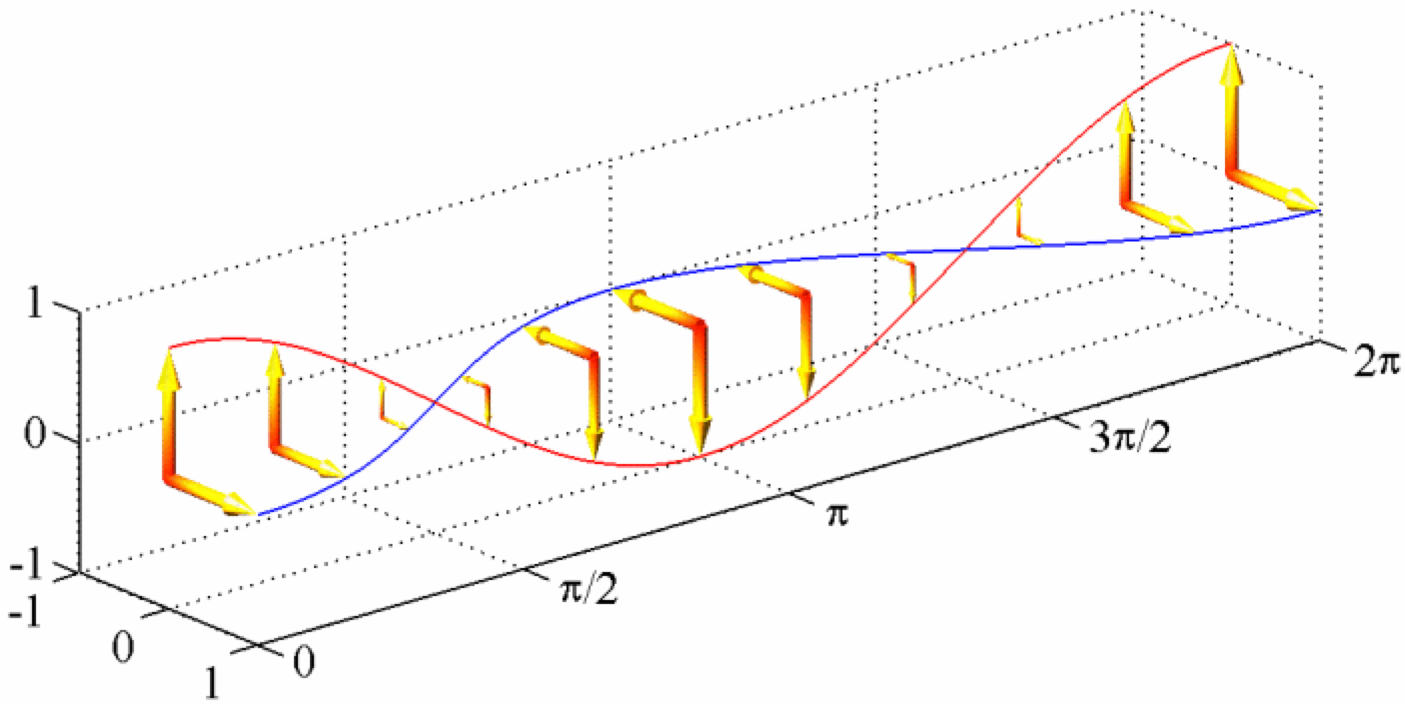
Licht – eine elektromagnetische Welle



Wir brauchen: Crash-Course in Elektrodynamik!

Das elektromagnetische Feld

Elektrisches Feld $\mathbf{E}$	Magnetisches Feld $\mathbf{B}$
Elektrische Ladung q	Elektrischer Strom I
Elektrische Ladungsdichte ρ	Elektrische Stromdichte $\mathbf{j}$
Elektrische Polarisation $\mathbf{P}$	Magnetisierung $\mathbf{M}$
Elektrische Suszeptibilität χ_e	Magnetische Suszeptibilität χ_m
Dielektrizitätskonstante ε	Permeabilität μ

Gaußsches Gesetz

Kraft zwischen zwei Ladungen $\mathbf{F} = \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$

Kraft von vielen Ladungen auf eine Probenladung:

$$\mathbf{F} = \sum_j \frac{q_j q}{r_j^2}\hat{\mathbf{r}}_j = q\mathbf{E}$$

Elektrisches Feld: $\mathbf{E} = \sum_j \frac{q_j}{r_j^2}\hat{\mathbf{r}}_j$

Gaußsches Gesetz

Elektrisches Feld für kontinuierliche Ladungsverteilung:

$$\mathbf{E} = \int \frac{\hat{\mathbf{r}}}{r^2} \rho(\mathbf{r}) dV$$

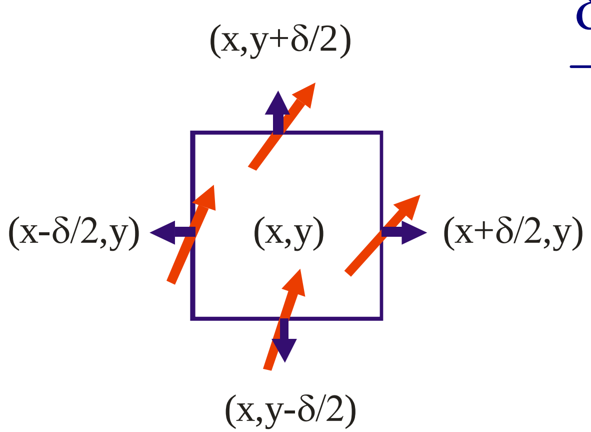
Fluß des elektrischen Feldes einer Ladung durch geschlossene Kugeloberfläche konstant

$$\int_A d\mathbf{A} \cdot \mathbf{E} = \frac{q}{r^2} \int_A dA = 4\pi q$$

Elektrisches Feld im ladungsfreien Raum divergenzfrei

Divergenz eines Vektorfeldes

$$\mathbf{div}\,\mathbf{E}(x,y,z) \sim \frac{\delta^2\left[E_x(x+\delta/2,y,z)-E_x(x-\delta/2,y,z)\right]}{\delta^3}+\frac{\delta^2\left[E_y(x,y+\delta/2,z)-E_y(x,y-\delta/2,z)\right]}{\delta^3}+\frac{\delta^2\left[E_z(x,y,z+\delta/2)-E_z(x,y,z-\delta/2)\right]}{\delta^3}$$



$$\boxed{\mathbf{div}\,\mathbf{E} = \frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}}$$

Gaußsches Gesetz

$$\int_A d\mathbf{A}\cdot\mathbf{E} = \frac{q}{r^2}\int_A dA = 4\pi q$$

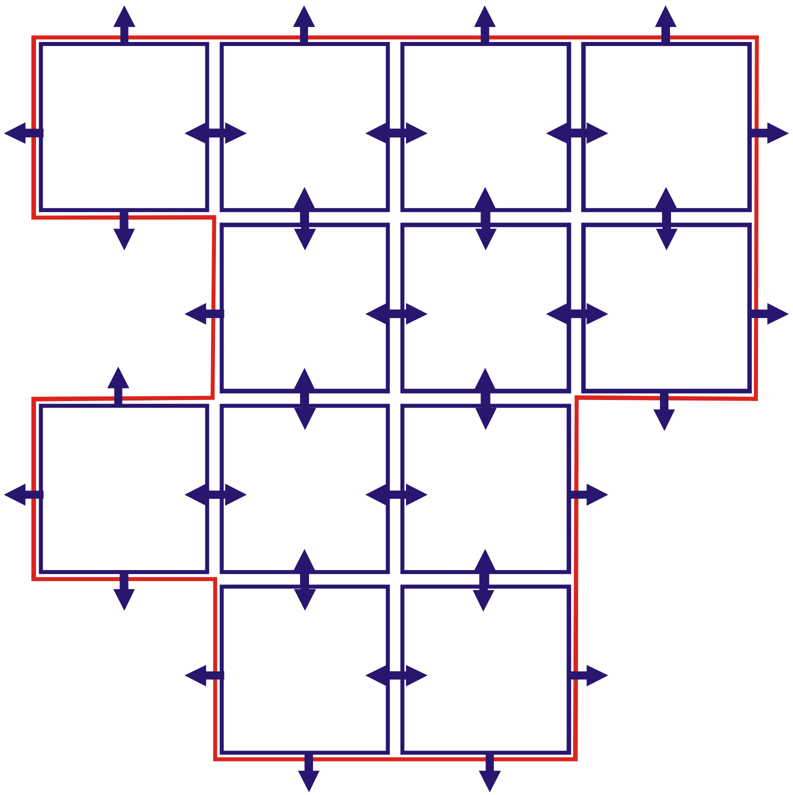
$$\mathbf{div}\,\mathbf{E} = 0$$ im ladungsfreien Bereich

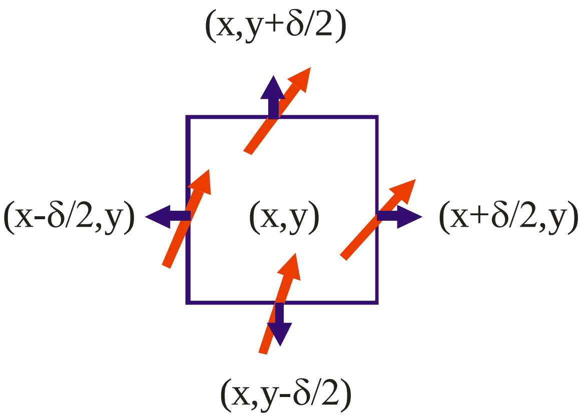
$$\mathbf{div}\,\mathbf{E} \sim \frac{4\pi q}{dV}$$ am Punkt der Ladung

$$\boxed{\mathbf{div}\,\mathbf{E} = 4\pi\rho}$$

Gaußsche Formel

Volumenintegral über den Divergenz eines Vektorfeldes





$$\int_V dV\, \mathbf{div}\, \mathbf{E} = \int_A d\mathbf{A}\cdot\mathbf{E}$$

Gaußsches Gesetz

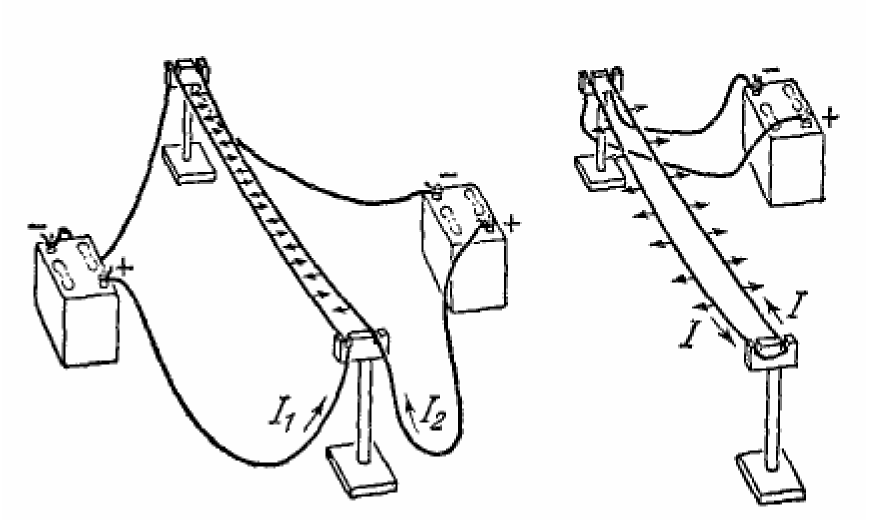
$$\boxed{\mathbf{div}\,\mathbf{E} = 4\pi\rho}$$

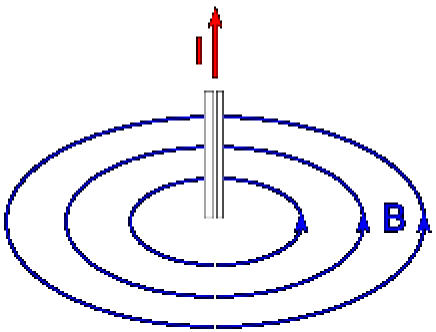
$$\int_V dV\,\mathbf{div}\,\mathbf{E} = 4\pi\int_V dV\rho$$

$$\int_A d\mathbf{A}\cdot\mathbf{E} = 4\pi Q$$

$$\mathbf{E} = \int \frac{\hat{\mathbf{r}}}{r^2}\rho(\mathbf{r})\,dV$$

Ampèresches Gesetz

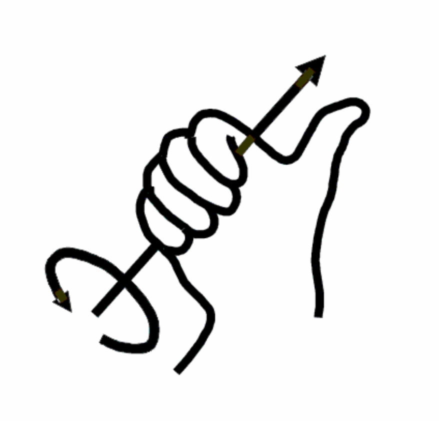


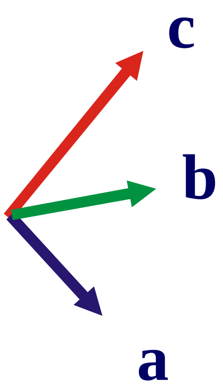


Kraft pro Länge zwischen zwei Stromleitern: $F = \frac{2 I_1 I_2}{r c^2}$

Magnetfeld: $d\mathbf{F} = \frac{I}{c} d\mathbf{l} \times \mathbf{B}$

Kreuzprodukt und Rechte-Hand-Regel



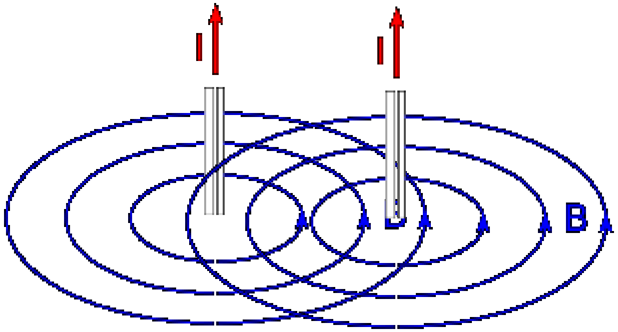


$$\mathbf{c} = \mathbf{a} \times \mathbf{b}$$

$$|\mathbf{c}| = |\mathbf{a}||\mathbf{b}|\sin\left(\widehat{\mathbf{a},\mathbf{b}}\right)$$

Ampèresches Gesetz

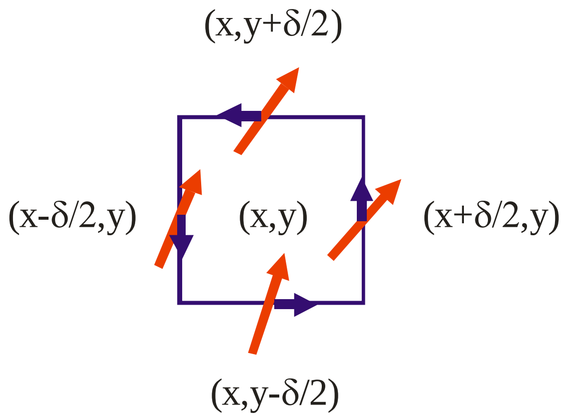
Magnetfeld: $$d\mathbf{F} = \frac{I}{c} d\mathbf{l} \times \mathbf{B}$$



$$\boxed{\mathrm{rot}\,\mathbf{B} = \frac{4\pi}{c}\mathbf{j}}$$

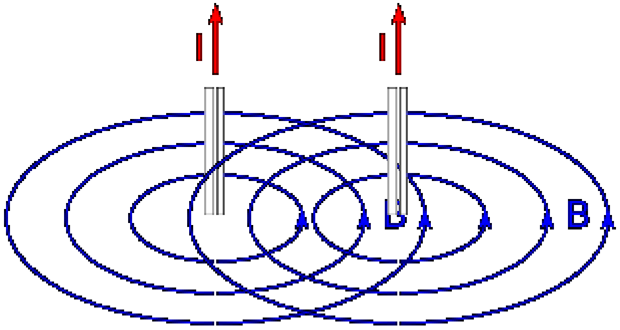
Rotor eines Vektorfeldes

$$\left[\mathbf{rot}\,\mathbf{B}(x,y,z)\right]_z \sim \frac{\delta\left[B_x(x,y-\delta/2,z)-B_x(x,y+\delta/2,z)\right]}{\delta^2}+\frac{\delta\left[B_y(x+\delta/2,y,z)-B_y(x,y-\delta/2,z)\right]}{\delta^2}$$



$$\boxed{\left[\mathbf{rot}\,\mathbf{B}\right]_z = \frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}}$$

Ampèresches Gesetz



Magnetfeld: $d\mathbf{F} = \frac{I}{c} d\mathbf{l} \times \mathbf{B}$

$$\mathbf{rot}\,\mathbf{B} = \frac{4\pi}{c}\mathbf{j}$$

Lorentz-Kraft: $$\mathbf{F} = \frac{1}{c}\mathbf{j} \times \mathbf{B}$$

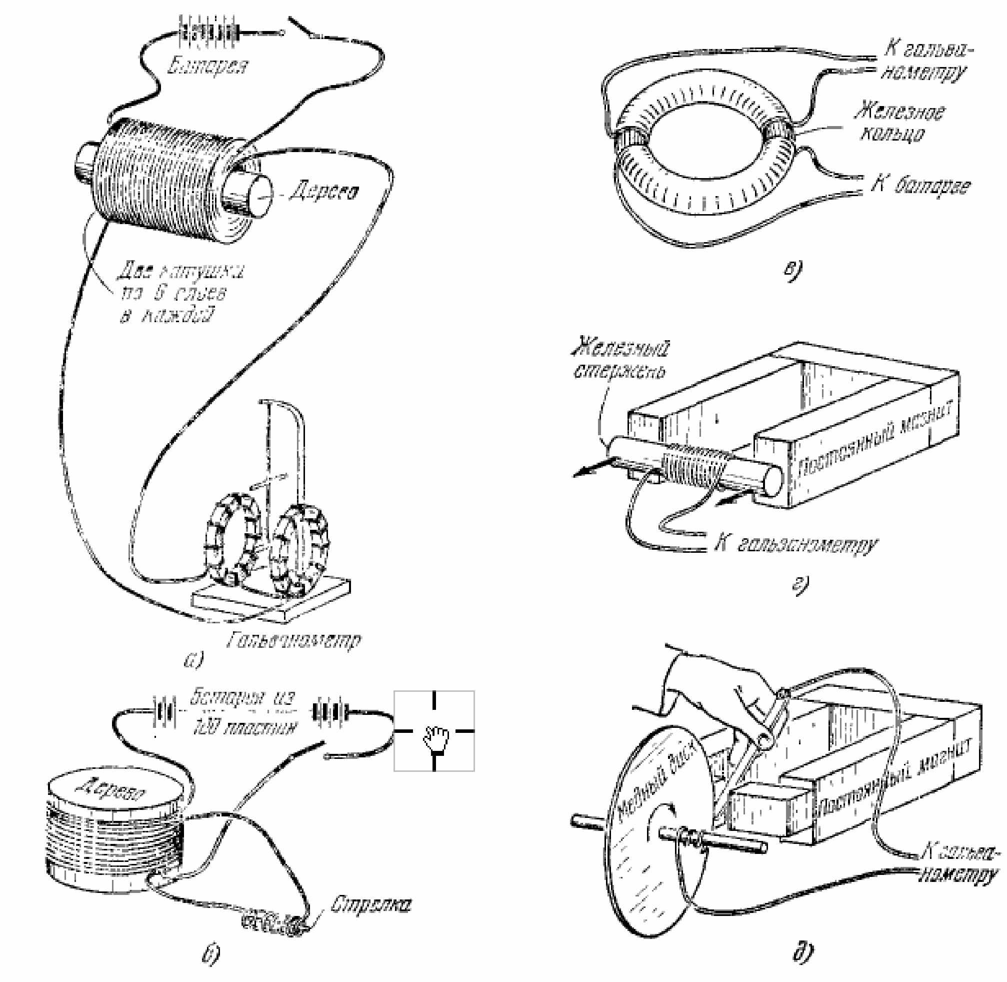
Zusammenfassung Gauß + Ampère

$$\mathbf{div}\,\mathbf{E} = 4\pi\rho$$

$$\mathbf{rot}\,\mathbf{B} = \frac{4\pi}{c}\,\mathbf{j}$$

$$\mathbf{F} = q\mathbf{E} + \frac{1}{c}\,\mathbf{j}\times\mathbf{B}$$

Faradays Induktionsgesetz



Faradays Induktionsgesetz

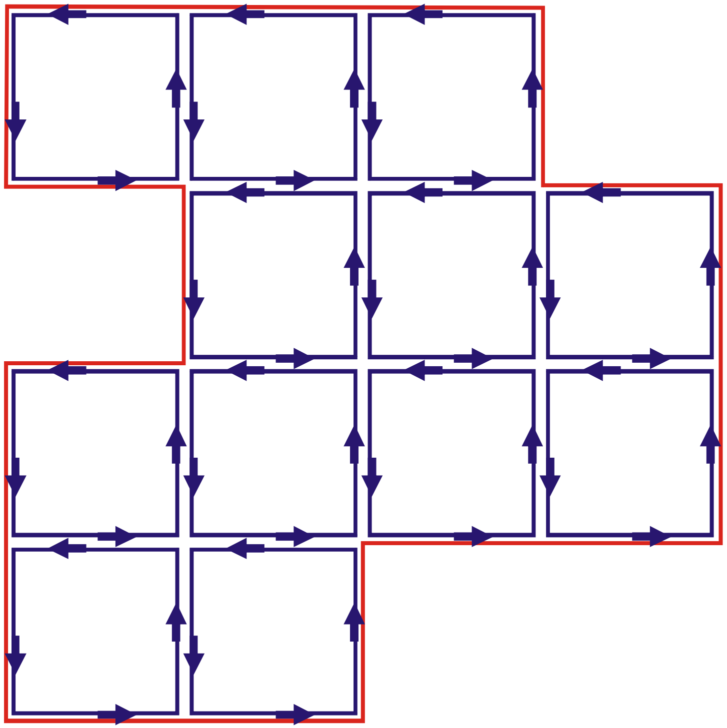
$$\underbrace{\text{Variables magnetisches Feld}}_{\frac{\partial \mathbf{B}}{\partial t}} \text{ induziert } \underbrace{\text{elektrisches Feld}}_{\mathbf{E}}$$

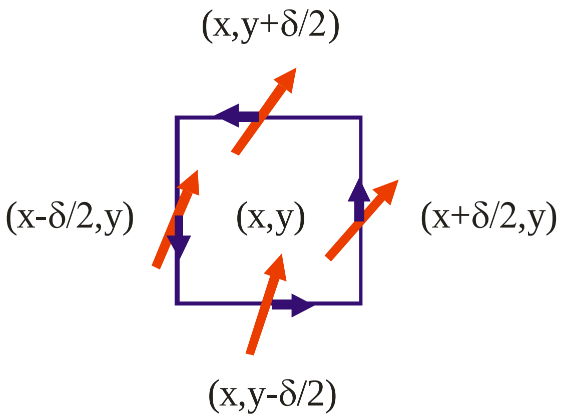
$$\boxed{\mathbf{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}}$$

$$\int_A d\mathbf{A}\cdot\mathbf{rot}\,\mathbf{E} = -\frac{1}{c}\int_A d\mathbf{A}\cdot\frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{c}\frac{\partial}{\partial t}\int_A d\mathbf{A}\cdot\mathbf{B} = -\frac{1}{c}\frac{\partial \Phi}{\partial t}$$

Stokessche Formel

Flächenintegral über den Rotor eines Vektorfeldes





$$\int_A d\mathbf{A} \cdot \mathbf{rot}\,\mathbf{E} = \int_l d\mathbf{l} \cdot \mathbf{E}$$

Faradays Induktionsgesetz

$$\boxed{\mathbf{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}}$$

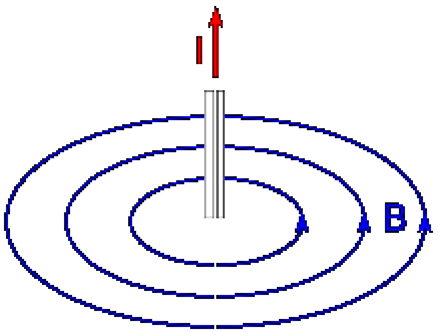
$$\int_A d\mathbf{A}\cdot\mathbf{rot}\,\mathbf{E} = -\frac{1}{c}\int_A d\mathbf{A}\cdot\frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{c}\frac{\partial}{\partial t}\int_A d\mathbf{A}\cdot\mathbf{B} = -\frac{1}{c}\frac{d\Phi}{dt}$$

$$\int_A d\mathbf{A}\cdot\mathbf{rot}\,\mathbf{E} = \int_l d\mathbf{l}\cdot\mathbf{E} = \Delta U$$

$$\Delta U = -\frac{1}{c}\frac{d\Phi}{dt}$$

Beispiel: Feld um elektrischen Leiter

$$\mathbf{rot}\,\mathbf{B} = \frac{4\pi}{c}\mathbf{j}$$



$$\iint_A d\mathbf{A}\cdot\mathbf{rot}\,\mathbf{B} = \iint_A d\mathbf{A}\cdot\frac{4\pi}{c}\mathbf{j}$$

$$\oint_{\partial A} d\mathbf{l}\cdot\mathbf{B} = \iint_A d\mathbf{A}\cdot\frac{4\pi}{c}\mathbf{j}$$

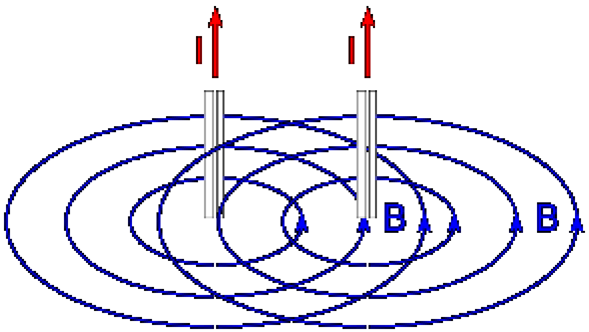
$$2\pi r B(r) = \frac{4\pi}{c} I$$

$$B(r) = \frac{2I}{cr}$$

Beispiel: Feld um elektrischen Leiter

$$\boxed{\textbf{rot}\,\mathbf{B} = \frac{4\pi}{c}\mathbf{j}} \Rightarrow B(r) = \frac{2I_1}{cr}$$

$$\boxed{\mathbf{F} = q\mathbf{E} + \frac{1}{c}\mathbf{j}\times\mathbf{B}}$$



$$F = \frac{IB}{c} = \frac{2I_1I_2}{c^2 r}$$

Beispiel: Feld um elektrischen Leiter

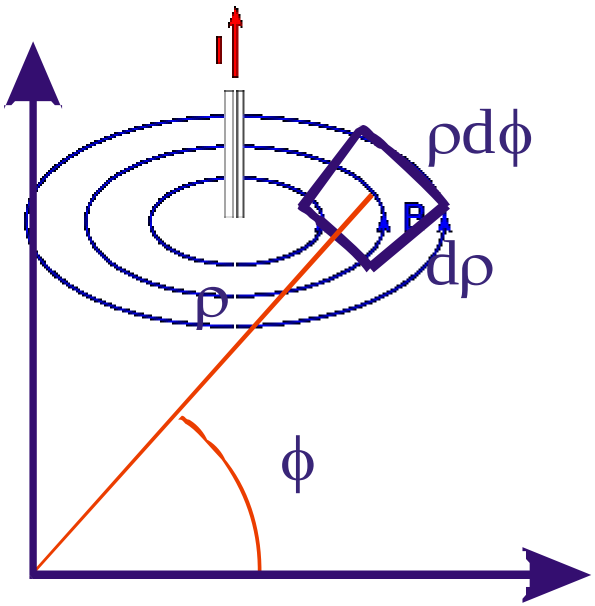
$$\boxed{\mathbf{rot}\,\mathbf{B} = \frac{4\pi}{c}\mathbf{j}} \rightarrow \mathbf{B} = \frac{2I}{cr}\mathbf{e}_\phi$$

$$\mathbf{B} \stackrel{?}{=} \frac{2I_1}{cr}\mathbf{e}_\phi + A(\rho)\mathbf{e}_\rho$$

$$\mathbf{rot}\left[A(\rho)\mathbf{e}_\rho\right] = 0$$

$$\mathbf{div}\left[A(\rho)\mathbf{e}_\rho\right] = \frac{1}{\rho}\frac{d}{d\rho}\left[\rho A(\rho)\right]$$

$$\boxed{\mathbf{div}\,\mathbf{B} = 0}$$



Zusammenfassung Gauß + Ampère + Faraday

$$\mathbf{div}\,\mathbf{E} = 4\pi\rho$$

$$\mathbf{rot}\,\mathbf{B} = \frac{4\pi}{c}\,\mathbf{j}$$

$$\mathbf{div}\,\mathbf{B} = 0$$

$$\mathbf{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\mathbf{F} = q\mathbf{E} + \frac{1}{c}\,\mathbf{j}\times\mathbf{B}$$