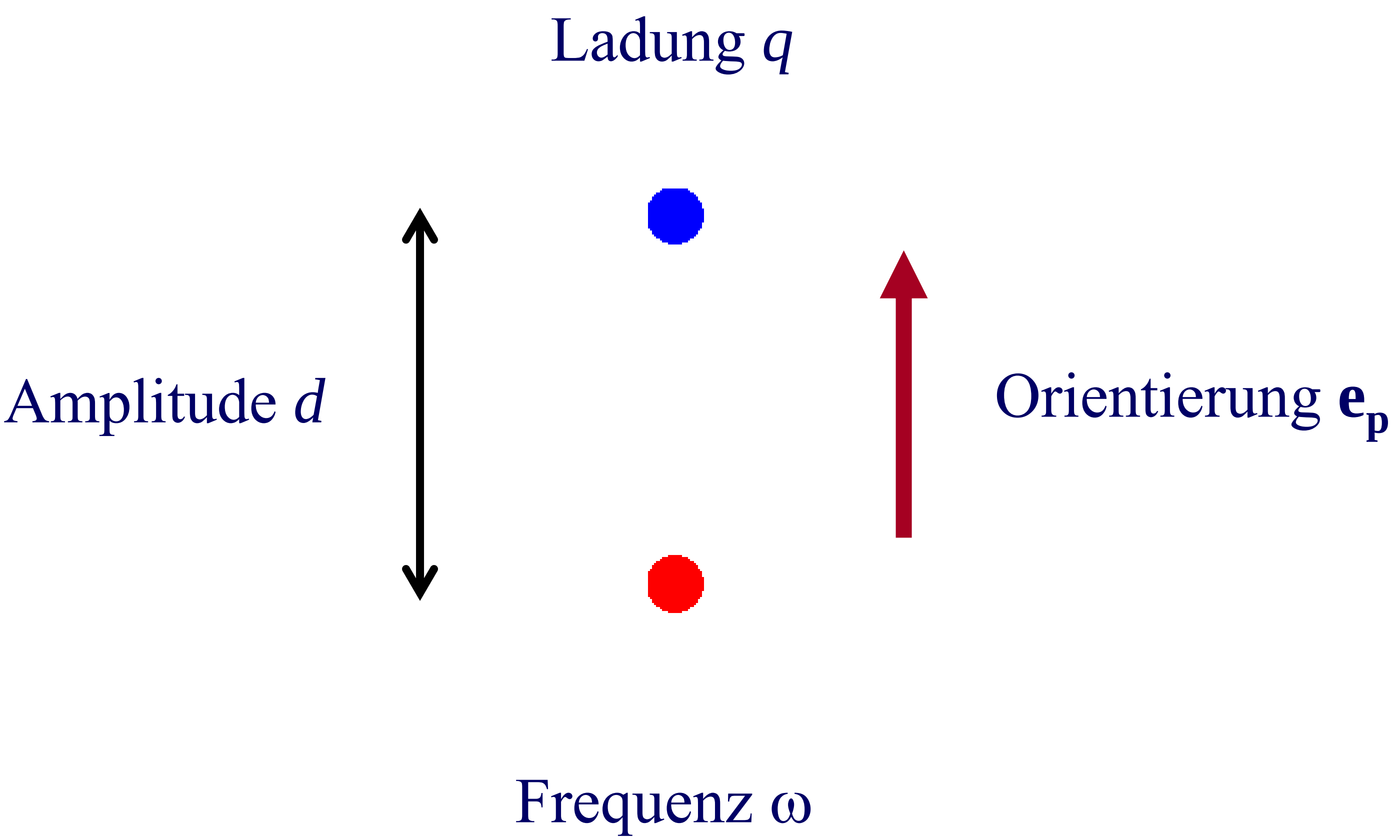
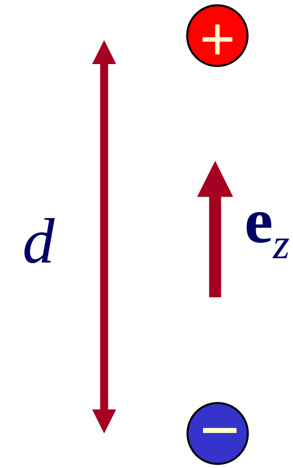
Emittierendes Molekül: Oszillierender Dipol



Oszillierender Dipol



$$z_{\pm} = \pm \frac{d}{2} e^{-i\omega t}$$

$$v_{\pm} = \frac{dz_{\pm}}{dt} = \mp i \frac{d}{2} \omega e^{-i\omega t}$$

$$j = -i \frac{qd}{2} \omega e^{-i\omega t} - i \frac{qd}{2} \omega e^{-i\omega t} = -ip\omega e^{-i\omega t}$$

Amplitude des Dipolmoments $p = qd$

$$\mathbf{j} = -i\omega \mathbf{p} e^{-i\omega t} \delta(\mathbf{r})$$

Oszillierender Dipol

$$\boxed{\mathbf{rot}\,\mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}}$$

$$\boxed{\mathbf{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}}$$

$$\mathbf{j} = -i\omega\mathbf{p}e^{-i\omega t}\delta(\mathbf{r})$$

$$\mathbf{rot}\,\mathbf{B} = -4\pi i k_0 \mathbf{p}\delta(\mathbf{r}) - ik_0\mathbf{E}$$

$$\mathbf{rot}\,\mathbf{E} = ik_0\mathbf{B}$$

$$k_0 = \frac{\omega}{c}$$

$$\boxed{\mathbf{div}\,\mathbf{E} = 4\pi\rho}$$

$$\boxed{\mathbf{div}\,\mathbf{B} = 0}$$

Potentiale:

$$\boxed{\mathbf{B} = \mathbf{rot}\,\mathbf{A}}$$

$$\boxed{\mathbf{E} = -\mathbf{grad}\,\phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}}$$

Oszillierender Dipol

$$\mathbf{rot}\,\mathbf{B} = -4\pi i k_0 \mathbf{p}\delta(\mathbf{r}) - i k_0 \mathbf{E} \qquad \mathbf{rot}\,\mathbf{E} = i k_0 \mathbf{B} \qquad k_0 = \frac{\omega}{c}$$

$$\mathbf{B} = \mathbf{rot}\,\mathbf{A} \qquad \mathbf{E} = -\mathbf{grad}\,\phi + i k_0 \mathbf{A}$$

$$\mathbf{rot}\,\mathbf{rot}\,\mathbf{A} = -4\pi i k_0 \mathbf{p}\delta(\mathbf{r}) - i k_0 \mathbf{grad}\,\phi + k_0^2 \mathbf{A}$$

$$\boxed{\mathbf{rot}\,\mathbf{rot}\,\mathbf{A} = \mathbf{grad}\,\mathbf{div}\,\mathbf{A} - \Delta\mathbf{A}}$$

$$\boxed{\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$

Laplace-Operator

Oszillierender Dipol

$$\mathbf{rot}\,\mathbf{B} = -4\pi i k_0 \mathbf{p}\delta(\mathbf{r}) - i k_0 \mathbf{E} \qquad \mathbf{rot}\,\mathbf{E} = i k_0 \mathbf{B} \qquad k_0 = \frac{\omega}{c}$$

$$\mathbf{B} = \mathbf{rot}\,\mathbf{A} \qquad \mathbf{E} = -\mathbf{grad}\,\phi + i k_0 \mathbf{A}$$

$$\mathbf{rot}\,\mathbf{rot}\,\mathbf{A} = -4\pi i k_0 \mathbf{p}\delta(\mathbf{r}) + i k_0 \mathbf{grad}\,\phi + k_0^2 \mathbf{A}$$

$$-\Delta\mathbf{A} - k_0^2 \mathbf{A} = -4\pi i k_0 \mathbf{p}\delta(\mathbf{r}) + \cancel{\mathbf{grad}\left(i k_0 \phi - \mathbf{div}\,\mathbf{A}\right)}$$

Eichfreiheit der Potentiale

$$\phi = -i k_0^{-1} \mathbf{div}\,\mathbf{A}$$

$$\Delta\mathbf{A} + k_0^2 \mathbf{A} = 4\pi i k_0 \mathbf{p}\delta(\mathbf{r})$$

Oszillierender Dipol

$$\Delta\mathbf{A} + k_0^2\mathbf{A} = 4\pi i k_0 \mathbf{p}\delta(\mathbf{r})$$

Was wäre, wenn $\Delta\mathbf{A} = 4\pi i k_0 \mathbf{p}\delta(\mathbf{r})$?

Vergleiche mit Elektrostatik:

$$\mathbf{div}\,\mathbf{E} = 4\pi\rho = -\mathbf{div}\,\mathbf{grad}\,\phi = -\Delta\phi$$

$$\mathbf{E}(\mathbf{r}) = \int \frac{\hat{\mathbf{R}}}{R^2}\rho(\mathbf{r}')\,dV' \longrightarrow \phi(\mathbf{r}) = \int \frac{1}{R}\rho(\mathbf{r}')\,dV'$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}'$$

$$\mathbf{A} = -\int \frac{1}{R} i k_0 \mathbf{p}\delta(\mathbf{r}')\,dV' = -\frac{i k_0 \mathbf{p}}{r}$$

Oszillierender Dipol

$$\Delta\mathbf{A} + k_0^2\mathbf{A} = 4\pi i k_0 \mathbf{p}\delta(\mathbf{r})$$

$$\Downarrow$$

$$\boxed{\mathbf{A} = -\frac{ik_0\mathbf{p}}{r}e^{ik_0 r}}$$

$$\Delta\mathbf{A} = 4\pi i k_0 \mathbf{p}\delta(\mathbf{r})$$

$$\Downarrow$$

$$\mathbf{A} = -\frac{ik_0\mathbf{p}}{r}$$

$$\mathbf{B} = \mathbf{rot}\,\mathbf{A} \qquad \mathbf{E} = -\mathbf{grad}\,\phi + ik_0\mathbf{A} \qquad \phi = -ik_0^{-1}\mathbf{div}\,\mathbf{A}$$

$$\mathbf{E} = ik_0^{-1}\mathbf{grad}\,\mathbf{div}\mathbf{A} + ik_0\mathbf{A}$$

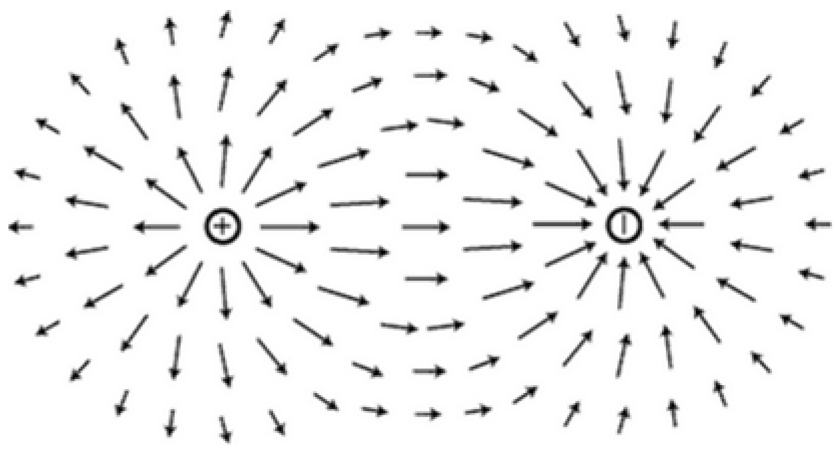
$$\boxed{\mathbf{E} = \left\{\frac{k_0^2}{r}\left[\mathbf{p} - \mathbf{e}_r(\mathbf{e}_r\cdot\mathbf{p})\right] + \left(\frac{ik_0}{r^2} - \frac{1}{r^3}\right)\left[\mathbf{p} - 3\mathbf{e}_r(\mathbf{e}_r\cdot\mathbf{p})\right]\right\}e^{ik_0 r - i\omega t}}$$

Oszillierender Dipol

$$\mathbf{E} = \left\{ \frac{k_0^2}{r} \left[\mathbf{p} - \mathbf{e}_r \left(\mathbf{e}_r \cdot \mathbf{p} \right) \right] + \left(\frac{ik_0}{r^2} - \frac{1}{r^3} \right) \left[\mathbf{p} - 3\mathbf{e}_r \left(\mathbf{e}_r \cdot \mathbf{p} \right) \right] \right\} e^{ik_0 r - i\omega t}$$

Im Grenzwert $k_0 \to 0$:

$$\mathbf{E}_{stat} = \frac{3\mathbf{e}_r \left(\mathbf{e}_r \cdot \mathbf{p} \right) - \mathbf{p}}{r^3}$$



Statischer elektrischer Dipol

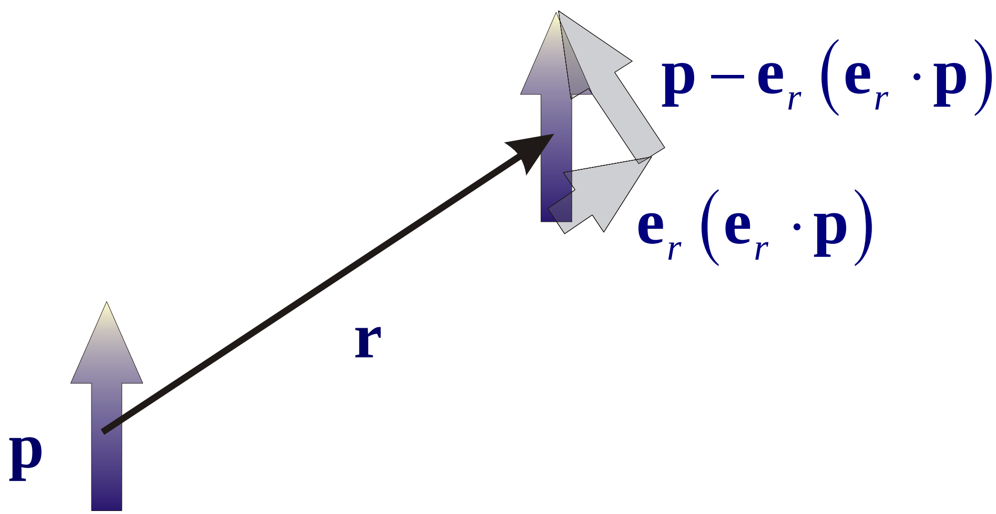
$$\mathbf{E}_{stat} = \mathbf{grad} \left(\mathbf{p} \cdot \mathbf{grad} \frac{1}{r} \right)$$

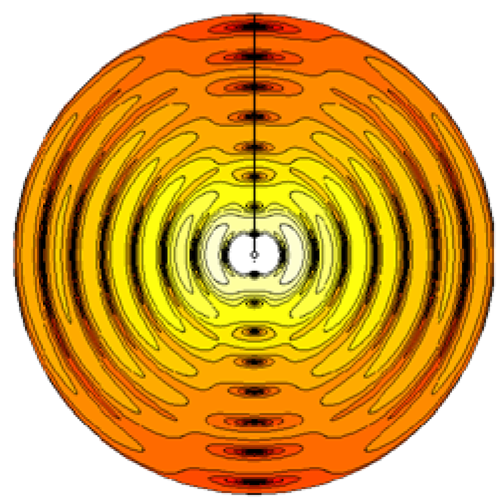
Oszillierender Dipol

$$\mathbf{E} = \left\{ \frac{k_0^2}{r} \left[\mathbf{p} - \mathbf{e}_r \left(\mathbf{e}_r \cdot \mathbf{p} \right) \right] + \left(\frac{ik_0}{r^2} - \frac{1}{r^3} \right) \left[\mathbf{p} - 3\mathbf{e}_r \left(\mathbf{e}_r \cdot \mathbf{p} \right) \right] \right\} e^{ik_0 r - i\omega t}$$

Fernfeld

Nahfeld





Energiebilanz des EM-Feldes

$$\mathbf{j}\cdot\mathbf{E}+W=0$$

$$\mathbf{j}\cdot\mathbf{E}=-\mathbf{E}\cdot\left(\frac{c}{4\pi}\,\mathbf{rot}\,\mathbf{B}-\frac{1}{4\pi}\frac{\partial\mathbf{E}}{\partial t}\right)$$

$$=-\frac{c}{4\pi}\mathbf{E}\cdot\mathbf{rot}\,\mathbf{B}-\frac{1}{8\pi}\frac{\partial E^2}{\partial t}$$

$$\mathbf{E}\cdot\mathbf{rot}\,\mathbf{B}=E_x\left(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}\right)+E_y\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)+E_z\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)$$

$$=\left(\frac{\partial\left(E_xB_z\right)}{\partial y}-B_z\frac{\partial E_x}{\partial y}-\frac{\partial\left(E_xB_y\right)}{\partial z}+B_y\frac{\partial E_x}{\partial z}\right)+\ldots$$

Energiebilanz des EM-Feldes

$$\mathbf{j}\cdot\mathbf{E}+W=0 \qquad \mathbf{j}\cdot\mathbf{E}=-\mathbf{E}\cdot\left(\frac{c}{4\pi}\,\mathbf{rot}\,\mathbf{B}-\frac{1}{4\pi}\frac{\partial\mathbf{E}}{\partial t}\right)$$

$$\mathbf{E}\cdot\mathbf{rot}\,\mathbf{B}=E_x\left(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}\right)+E_y\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)+E_z\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)$$

$$=\left(\frac{\partial\left(E_xB_z\right)}{\partial y}-B_z\frac{\partial E_x}{\partial y}-\frac{\partial\left(E_xB_y\right)}{\partial z}+B_y\frac{\partial E_x}{\partial z}\right)+\ldots$$

$$=\mathbf{div}\left(\mathbf{B}\times\mathbf{E}\right)+\mathbf{B}\cdot\mathbf{rot}\,\mathbf{E}$$

$$=\mathbf{div}\left(\mathbf{B}\times\mathbf{E}\right)-\mathbf{B}\cdot\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=-\mathbf{div}\left(\mathbf{E}\times\mathbf{B}\right)-\frac{1}{2c}\frac{\partial B^2}{\partial t}$$

Energiebilanz des EM-Feldes

$$\mathbf{j}\cdot\mathbf{E} + W = 0 \qquad \mathbf{j}\cdot\mathbf{E} = -\mathbf{E}\cdot\left(\frac{c}{4\pi}\,\mathbf{rot}\,\mathbf{B} - \frac{1}{4\pi}\frac{\partial\mathbf{E}}{\partial t}\right)$$

$$\mathbf{E}\cdot\mathbf{rot}\,\mathbf{B} = -\mathbf{div}\left(\mathbf{E}\times\mathbf{B}\right) - \frac{1}{2c}\frac{\partial B^2}{\partial t}$$

$$\mathbf{j}\cdot\mathbf{E} = -\mathbf{div}\underbrace{\left(\frac{c}{4\pi}\mathbf{E}\times\mathbf{B}\right)}_{\text{Energieflußdichte}} - \frac{\partial}{\partial t}\underbrace{\left(\frac{E^2+B^2}{8\pi}\right)}_{\text{Energiedichte}}$$

$$\boxed{\mathbf{P} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B}}$$

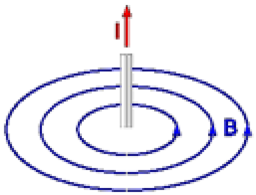
Poynting-Vektor

$$\dot{\mathcal{E}} + \mathbf{div}\,\mathbf{P} = -\mathbf{j}\cdot\mathbf{E}$$

Energieabstrahlung des Dipols

$$\mathbf{E} = \left\{ \frac{k_0^2}{r}\left[\mathbf{p} - \mathbf{e}_r\left(\mathbf{e}_r \cdot \mathbf{p}\right)\right] + \left(\frac{ik_0}{r^2} - \frac{1}{r^3}\right)\left[\mathbf{p} - 3\mathbf{e}_r\left(\mathbf{e}_r \cdot \mathbf{p}\right)\right] \right\} e^{ik_0 r - i\omega t}$$

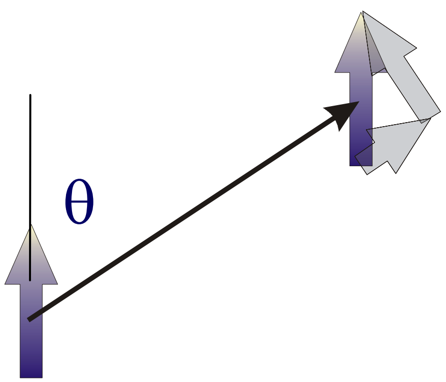
$$\mathbf{B} = \left(\mathbf{p} \times \mathbf{e}_r\right)\left(\frac{k_0^2}{r} + \frac{ik_0}{r^2}\right) e^{ik_0 r - i\omega t}$$



$$\mathbf{P} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}$$

$$\mathbf{P} = \frac{c}{8\pi}\mathrm{Re}\left(\mathbf{E} \times \mathbf{B}^*\right) \underset{r\to\infty}{=} \mathbf{e}_r \frac{cp^2}{8\pi}\frac{k_0^4}{r^2}\sin^2\theta$$

$$S_{Dipol} = r^2 \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi\, P = \frac{1}{3} c p^2 k_0^4$$

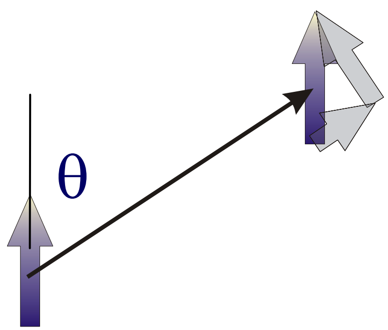


Zusammenfassung Dipolemission

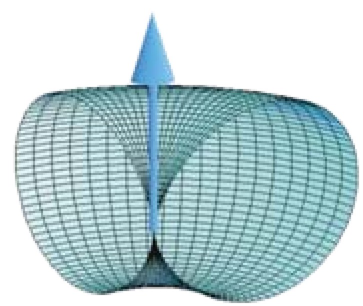
$$\mathbf{E} = \left\{ \frac{k_0^2}{r}\left[\mathbf{p} - \mathbf{e}_r\left(\mathbf{e}_r \cdot \mathbf{p}\right)\right] + \left(\frac{ik_0}{r^2} - \frac{1}{r^3}\right)\left[\mathbf{p} - 3\mathbf{e}_r\left(\mathbf{e}_r \cdot \mathbf{p}\right)\right]\right\} e^{ik_0 r - i\omega t}$$

$$\mathbf{B} = \left(\mathbf{p} \times \mathbf{e}_r\right)\left(\frac{k_0^2}{r} + \frac{ik_0}{r^2}\right) e^{ik_0 r - i\omega t}$$

$$\mathbf{P} \underset{r\to\infty}{=} \mathbf{e}_r \frac{cp^2}{8\pi}\frac{k_0^4}{r^2}\sin^2\theta$$



$$S_{Dipol} = \frac{1}{3}cp^2 k_0^4$$



Lebenszeit eines angeregten Moleküls

Energie eines harmonischen Oszillators der Masse *m*, Oszillationskreisfrequenz ω und Oszillationsamplitude *d:*

$$E = \frac{1}{2} m\omega^2 d^2$$

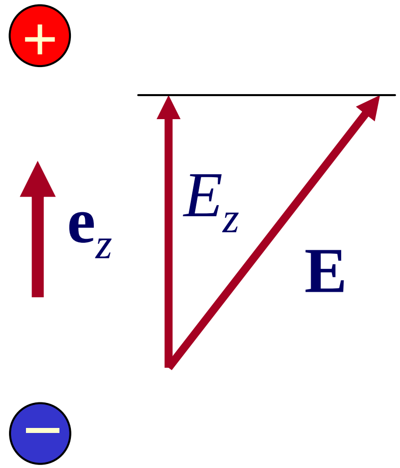
Energieabstrahlung je Zeiteinheit:

$$S_{Dipol} = \frac{1}{3} \frac{p^2\omega^4}{c^3}$$

Abschätzung der Lebenszeit:

$$\tau = \frac{E}{S_{Dipol}} = \frac{3mc^3}{2q^2 d^2 \omega^2} \quad \xrightarrow[\text{@ 500 nm}]{\text{für Elektron}} \quad \tau = 11.25\ \text{ns}$$

Dipolabsorption, Energie der ebenen Welle



Absorption proportional zu

$$\boxed{\left|\mathbf{E}\cdot\mathbf{e}_\mathbf{p}\right|^2}$$

Energiestromdichte der ebenen Welle:

$$\boxed{\mathbf{P} = \frac{c}{8\pi}\operatorname{Re}\left(\mathbf{E}\times\mathbf{B}^*\right) = \frac{c}{8\pi}\left|\mathbf{E}_0\right|^2}$$