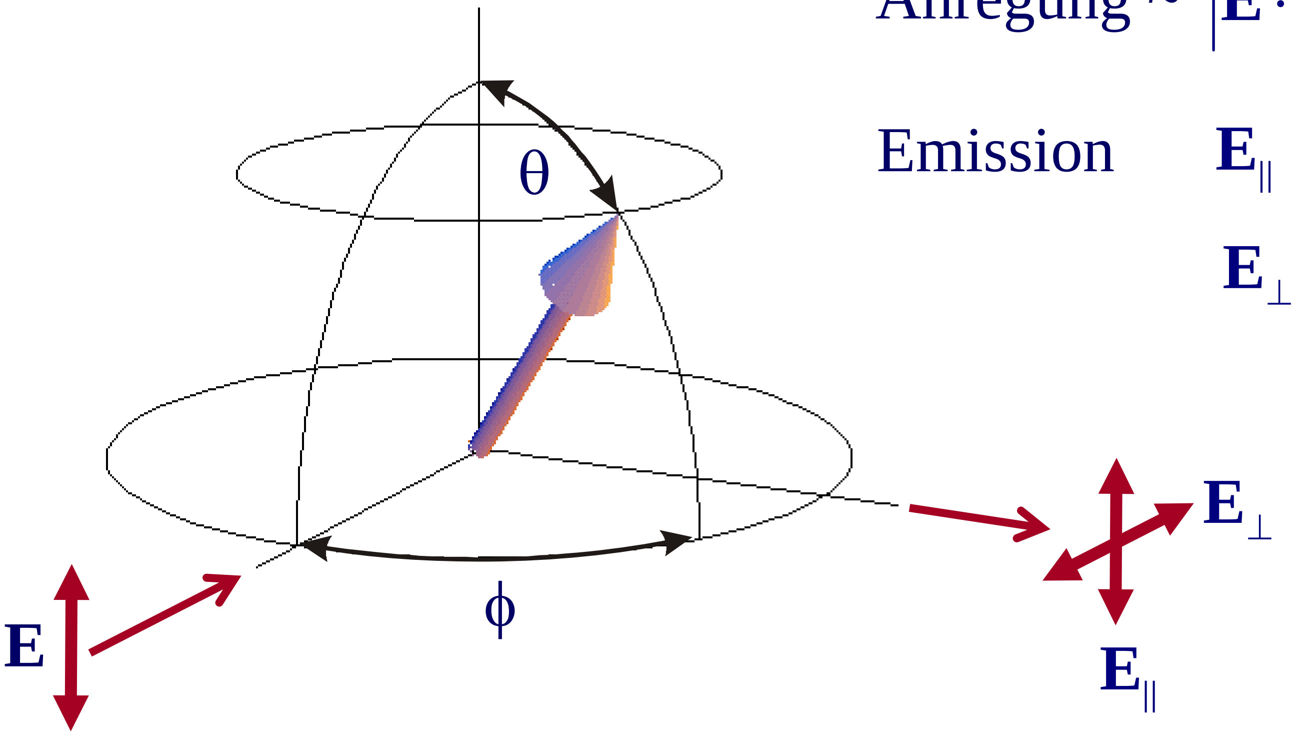
Fluoreszenzanisotropie

$$\text{Anregung} \sim \left|\mathbf{E}\cdot\mathbf{e_p}\right|^2 = E^2\cos^2\theta$$

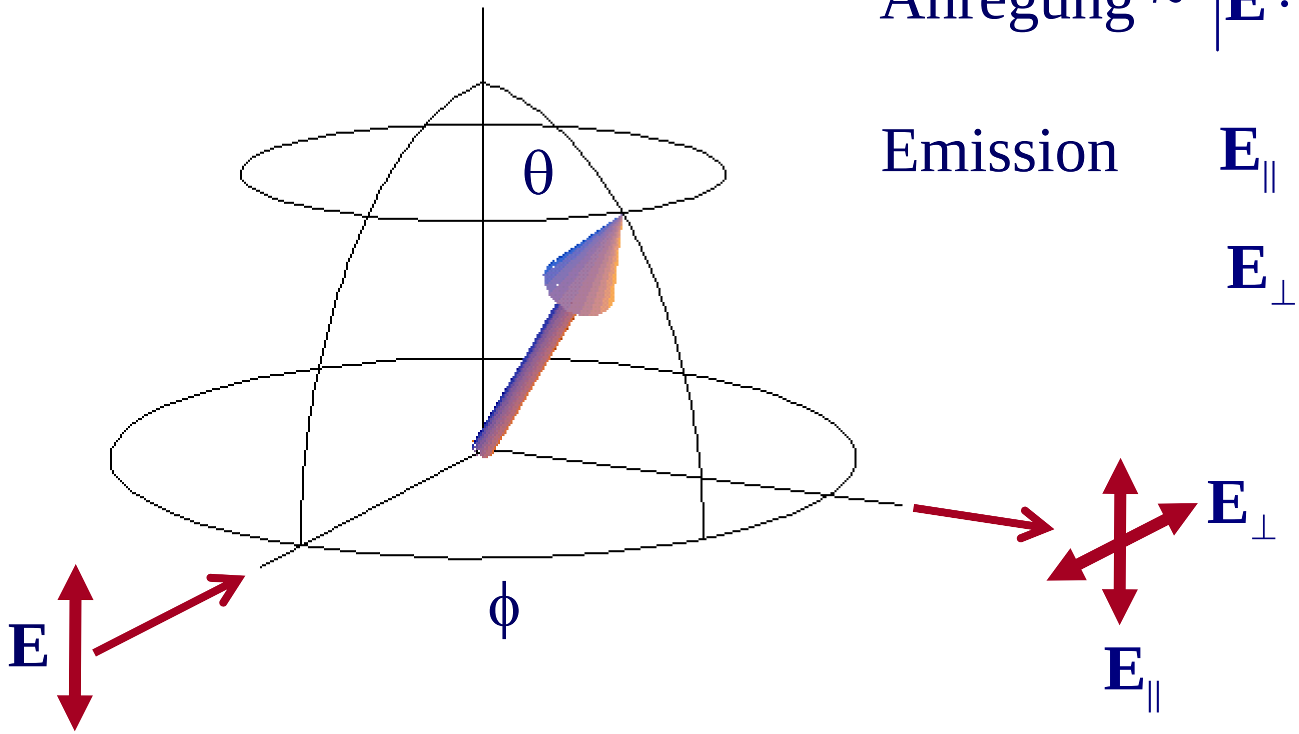
Emission $\quad \mathbf{E}_{\parallel} \sim \cos\theta$

$$\mathbf{E}_{\perp} \sim \sin\theta\cos\phi$$



Detektorsignal proportional zu Anregungseffizienz mal Energiestromdichte der Emission

Fluoreszenzanisotropie für fixe Dipole



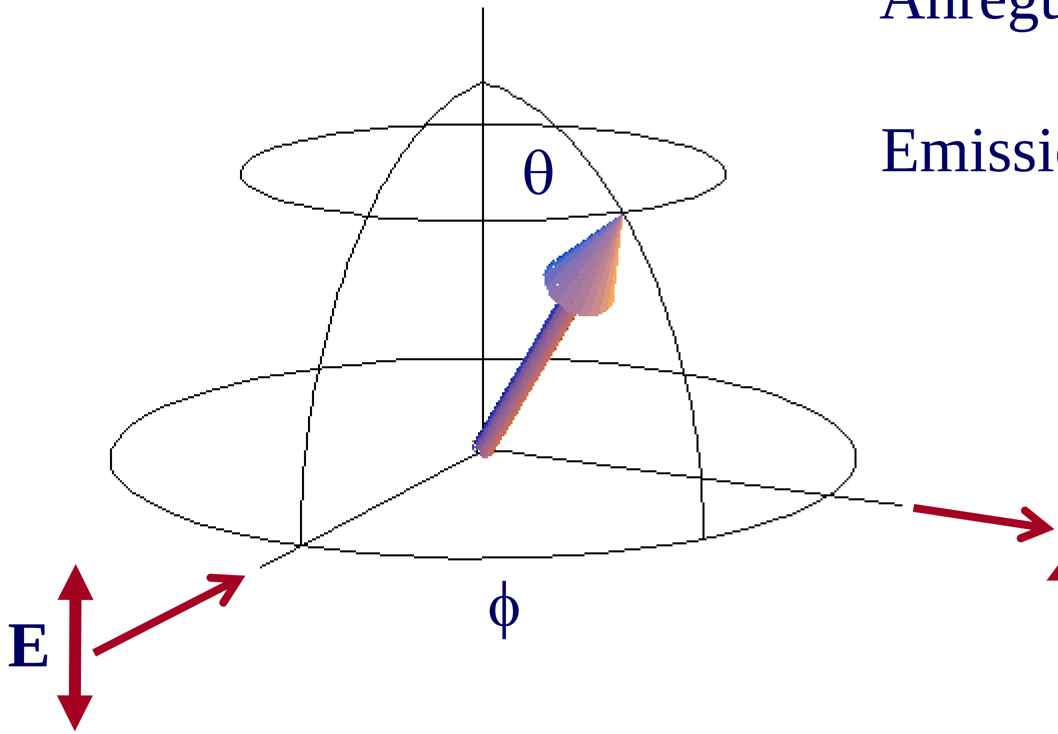
$$\text{Anregung} \sim \left|\mathbf{E}\cdot\mathbf{e_p}\right|^2 = E^2\cos^2\theta$$

$$\text{Emission} \qquad \mathbf{E}_{\|} \sim \cos\theta$$

$$\mathbf{E}_{\perp} \sim \sin\theta\cos\phi$$

$$I_{\|} \sim \int d\Omega \left|\mathbf{E}\cdot\mathbf{e}_{ex}\right|^2 \left|\mathbf{E}_{\|}\right|^2 \sim \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \cos^4\theta = \frac{4\pi}{5}$$

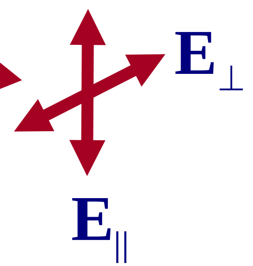
Fluoreszenzanisotropie für fixe Dipole



Anregung $\sim \left|\mathbf{E}\cdot\mathbf{e_p}\right|^2 = E^2\cos^2\theta$

Emission $\mathbf{E}_{||} \sim \cos\theta$

$\mathbf{E}_{\perp} \sim \sin\theta\cos\phi$



$$I_{\perp} \sim \int d\Omega \left|\mathbf{E}\cdot\mathbf{e}_{ex}\right|^2 \left|\mathbf{E}_{\perp}\right|^2 \sim \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \cos^2\theta \sin^2\theta \cos^2\phi = \frac{4\pi}{15}$$

Fluoreszenzanisotropie für fixe Dipole

$$I_{\parallel} \sim \int d\Omega \left|\mathbf{E}\cdot\mathbf{e}_{ex}\right|^2 \left|\mathbf{E}_{\parallel}\right|^2 \sim \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \cos^4\theta = \frac{4\pi}{5}$$

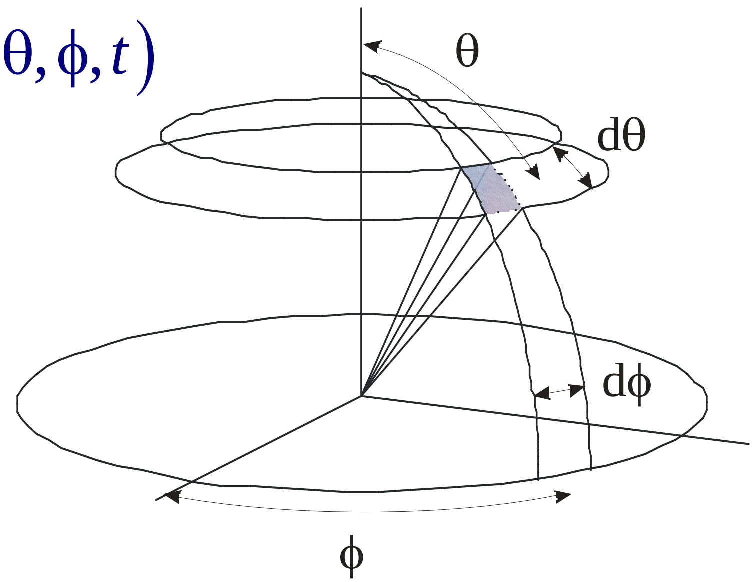
$$I_{\perp} \sim \int d\Omega \left|\mathbf{E}\cdot\mathbf{e}_{ex}\right|^2 \left|\mathbf{E}_{\perp}\right|^2 \sim \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \cos^2\theta \sin^2\theta \cos^2\phi = \frac{4\pi}{15}$$

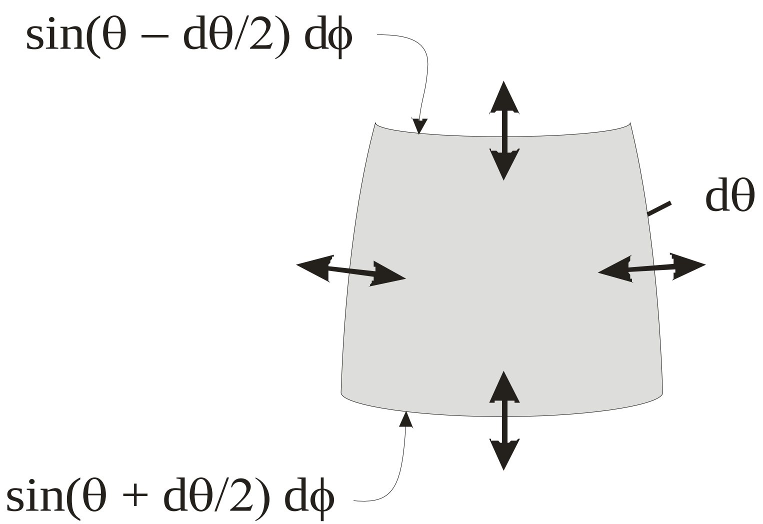
$$r = \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + 2I_{\perp}} = \frac{2}{5}$$

Isotrope Verteilung fixer Dipole

Rotationsdiffusion

$$P(\theta, \phi, t)$$





$$\frac{\partial P}{\partial t} + \mathbf{div}\,\mathbf{J} = 0$$

$$\mathbf{J} = -D\,\mathbf{grad}P$$

$$\mathbf{div}\,\mathbf{J} = \frac{1}{\sin\theta}\frac{\partial\left(\sin\theta J_\theta\right)}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial J_\phi}{\partial\phi}$$

$$\mathbf{grad}P = \mathbf{e}_\theta \frac{\partial P}{\partial\theta} + \mathbf{e}_\phi \frac{1}{\sin\theta}\frac{\partial P}{\partial\phi}$$

Rotationsdiffusion

$$\frac{\partial P(\theta,\phi,t)}{\partial t} = D\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial P(\theta,\phi,t)}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 P(\theta,\phi,t)}{\partial\phi^2}\right]$$

Anfangszustand: $\left|\mathbf{E}\cdot\mathbf{e_p}\right|^2 = E^2\cos^2\theta \longrightarrow P_0(\theta,\phi) = \frac{3}{4\pi}\cos^2\theta$

$$\frac{\partial P(\theta,t)}{\partial t} = D\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial P(\theta,t)}{\partial\theta}$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\cos^2\theta}{\partial\theta} = -\frac{2}{\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta\cos\theta = 2 - 6\cos^2\theta$$

Rotationsdiffusion

$$\frac{\partial P(\theta,t)}{\partial t} = D\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial P(\theta,t)}{\partial\theta} \qquad P_0(\theta,\phi) = \frac{3}{4\pi}\cos^2\theta$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\cos^2\theta}{\partial\theta} = -\frac{2}{\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta\cos\theta = 2-6\cos^2\theta$$

$$P(\theta,t) = A(t) + B(t)\cos^2\theta$$

$$\dot{A}(t) + \dot{B}(t)\cos^2\theta = D\left(2-6\cos^2\theta\right)B(t)$$

$$\dot{A}(t) = 2DB(t) \quad \rightarrow \quad A(t) = A_0 - B_0 e^{-6Dt}/3$$

$$\dot{B}(t) = -6DB(t) \quad \rightarrow \quad B(t) = B_0 e^{-6Dt}$$

Rotationsdiffusion

$$\frac{\partial P(\theta,t)}{\partial t} = D\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial P(\theta,t)}{\partial\theta} \qquad P_0(\theta,\phi) = \frac{3}{4\pi}\cos^2\theta$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\cos^2\theta}{\partial\theta} = -\frac{2}{\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta\cos\theta = 2-6\cos^2\theta$$

$$P(\theta,t) = A(t) + B(t)\cos^2\theta$$

$$A(t) = A_0 - B_0 e^{-6Dt}/3$$

$$B(t) = B_0 e^{-6Dt}$$

$$P(\theta,t) = \frac{1}{4\pi} + \frac{3}{4\pi}\left(\cos^2\theta - \frac{1}{3}\right)e^{-6Dt}$$